NAG C Library Function Document

nag_complex_polygamma (s14afc)

1 Purpose
nag_complex_polygamma (s14afc) returns the value of the kth derivative of the psi function \( \psi(z) \), for complex \( z \) and \( k = 0, 1, \ldots, 4 \).

2 Specification

```c
Complex nag_complex_polygamma(Complex z, Integer k, NagError *fail)
```

3 Description

This routine evaluates an approximation to the kth derivative of the psi function \( \psi(z) \) given by

\[
\psi^{(k)}(z) = \frac{d^k}{dz^k} \psi(z) = \frac{d^k}{dz^k} \left( \frac{d}{dz} \log \Gamma(z) \right),
\]

where \( z = x + iy \) is complex provided \( y \neq 0 \) and \( k = 0, 1, \ldots, 4 \). If \( y = 0 \), \( z \) is real and thus \( \psi^{(k)}(z) \) is singular when \( z = 0, -1, -2, \ldots \).

Note that \( \psi^{(k)}(z) \) is also known as the polygamma function. Specifically, \( \psi^{(0)}(z) \) is often referred to as the digamma function and \( \psi^{(1)}(z) \) as the trigamma function in the literature. Further details can be found in Abramowitz and Stegun (1972).

nag_complex_polygamma is based on a modification of the method proposed by Kölbig K S (1972).
To obtain the value of \( \psi^{(k)}(z) \) when \( z \) is real, nag_real_polygamma (s14aec) can be used.

4 Parameters

1: \( z \) – Complex

\( \text{Input} \)

On entry: the argument \( z \) of the function.

Constraint: \( z.\text{re} \) must not be ‘too close’ (see Section 5) to a non-positive integer when \( z.\text{im} = 0.0 \).

2: \( k \) – Integer

\( \text{Input} \)

On entry: the function \( \psi^{(k)}(z) \) to be evaluated.

Constraint: \( 0 \leq k \leq 4 \).

3: \( \text{fail} \) – NagError *

\( \text{Input/Output} \)

The NAG error parameter (see the Essential Introduction).

5 Error Indicators and Warnings

**NE_INT**
On entry, \( k = <\text{value}> \).

Constraint: \( 0 \leq k \leq 4 \).

**NE_COMPLEX**

On entry, \( z = (<\text{value}>, <\text{value}>), \)

Constraint: \( z.\text{re} \) must not be ‘too close’ to a non-positive integer when \( z.\text{im} = 0.0 \). That is, \( |z.\text{re} - \text{nint}(z.\text{re})| \geq \text{machine precision} \times |\text{nint}(z.\text{re})| \).
NE_OVERFLOW_LIKELY

The evaluation has been abandoned due to the likelihood of overflow. The result is returned as zero.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

6 Further Comments

6.1 Accuracy

Empirical tests have shown that the maximum relative error is a loss of approximately two decimal places of precision.

6.2 References

Kölbl K S (1972) Programs for computing the logarithm of the gamma function, and the digamma function, for complex arguments Comp. Phys. Comm. 4 221–226


7 See Also

None.

8 Example

The example program evaluates the psi (trigamma) function $\psi^{(1)}(z)$ at $z = -1.5 + 2.5i$, and prints the results.

8.1 Program Text

/* nag_complex_polygamma (sl4afc) Example Program.
 * Copyright 2000 Numerical Algorithms Group.
 * NAG C Library
 * Mark 6, 2000.
 */
#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Complex z, z__l;
    Integer exit_status=0;
    Integer k;
    NagError fail;

    INIT_FAIL(fail);
    Vprintf("sl4afc Example Program Results\n\n");
    /* Skip heading in data file */
    Vscanf("%*[\n ]");
8.2 Program Data

sl4afc Example Program Data
(1.2,5.0) 0
(0.5,-0.2) 1
(-1.5,2.5) 1
(8.0,3.3) 3
(2.9,7.5) 4 : Values of z and k

8.3 Program Results

sl4afc Example Program Results

<table>
<thead>
<tr>
<th>z</th>
<th>k</th>
<th>((D'K/D^2K)\psi(z))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2, 5.0</td>
<td>0</td>
<td>1.6176e+00, 1.4312e+00</td>
</tr>
<tr>
<td>0.5, -0.2</td>
<td>1</td>
<td>3.4044e+00, 2.5394e+00</td>
</tr>
<tr>
<td>-1.5, 2.5</td>
<td>1</td>
<td>-1.9737e-01, -2.4271e-01</td>
</tr>
<tr>
<td>8.0, 3.3</td>
<td>3</td>
<td>1.1814e-03, -3.4188e-03</td>
</tr>
<tr>
<td>2.9, 7.5</td>
<td>4</td>
<td>-5.0227e-04, -1.4955e-03</td>
</tr>
</tbody>
</table>