NAG C Library Function Document

nag_prob_non_central_students_t (g01gbc)

1 Purpose

nag_prob_non_central_students_t (g01gbc) returns the lower tail probability for the non-central Student’s t-distribution.

2 Specification

#include <nag.h>
#include <nag01.h>

double nag_prob_non_central_students_t (double t, double df, double delta, 
double tol, Integer max_iter, NagError *fail)

3 Description

The lower tail probability of the non-central Student’s t-distribution with \( \nu \) degrees of freedom and non-centrality parameter \( \delta \), \( P(T \leq t : \nu; \delta) \) is defined by:

\[
P(T \leq t : \nu; \delta) = C_\nu \int_0^\infty \left( \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\alpha - \delta} e^{-x^2/2} dx \right) u^{\nu-1} e^{-u^2/2} du, \quad \nu > 0.0
\]

with

\[
C_\nu = \frac{1}{\Gamma(\nu/2)} 2^{(\nu-2)/2}, \quad \alpha = \frac{t}{\sqrt{\nu}}
\]

The probability is computed in one of two ways,

(a) when \( t = 0.0 \), the relationship to the normal is used

\[
P(T \leq t : \nu; \delta) = \frac{1}{\sqrt{2\pi}} \int_{-\delta}^\infty e^{-u^2/2} du;
\]

(b) otherwise the series expansion described in Amos (1964) (equation 9) is used. This involves the sums of confluent hypergeometric functions, the terms of which are computed using recurrence relationships.

4 Parameters

1. \( t \) – double 
   \textbf{Input}
   \textit{On entry}: the deviate from the Student’s t-distribution with \( \nu \) degrees of freedom, \( t \).

2. \( df \) – double 
   \textbf{Input}
   \textit{On entry}: the degrees of freedom of the Student’s t-distribution, \( \nu \).
   \textit{Constraint}: \( df \geq 1.0 \).

3. \( delta \) – double 
   \textbf{Input}
   \textit{On entry}: the non-centrality parameter of the Students t-distribution, \( \delta \).
4: \( \text{tol} \) – double

\textit{Input}

\textit{On entry:} the absolute accuracy required by the user in the results.
If \texttt{nag\_prob\_non\_central\_students\_t} is entered with \texttt{tol} greater than or equal to 1.0 or less than \(10 \times \text{machine precision}\) (see \texttt{nag\_machine\_precision} (X02AJC)), then the value of \(10 \times \text{machine precision}\) is used instead.

5: \( \text{max\_iter} \) – Integer

\textit{Input}

\textit{On entry:} the maximum number of terms that are used in each of the summations.
\textit{Suggested value:} 100. See Section 6 for further comments.
\textit{Constraint:} \( \text{max\_iter} \geq 1 \).

6: \( \text{fail} \) – NagError *

\textit{Input/Output}

The NAG error parameter (see the Essential Introduction).

5 \ Error Indicators and Warnings

\textbf{NE\_REAL\_ARG\_LT}

\textit{On entry, df} must not be less than 1.0: \( df = <value> \).

\textbf{NE\_INT\_ARG\_LT}

\textit{On entry, max\_iter} must not be less than 1: \( max\_iter = <value> \).

\textbf{NE\_SERIES}

One of the series has failed to converge with \( df = <value> \) and \( max\_iter = <value> \). Reconsider the requested tolerance and/or the maximum number of iterations.

\textbf{NE\_PROBABILITY}

The probability is too small to calculate accurately.

\textbf{NE\_INTERNAL\_ERROR}

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

6 \ Further Comments

The rate of convergence of the series depends, in part, on the quantity: \( t^2/(t^2 + \nu) \). The smaller this quantity the faster the convergence. Thus for large \( t \) and small \( \nu \) the convergence may be slow. If \( \nu \) is an integer then one of the series to be summed is of finite length.

If two tail probabilities are required then the relationship of the \( t \)-distribution to the \( F \)-distribution can be used:

\[ F = T^2, \lambda = \delta^2, \nu_1 = 1 \text{ and } \nu_2 = \nu, \]

and a call made to \texttt{nag\_prob\_non\_central\_f\_dist} (g01gdc).

\textbf{Note:} this routine only allows degrees of freedom greater than or equal to 1 although values between 0 and 1 are theoretically possible.

6.1 \ Accuracy

The series described in Amos (1964) are summed until an estimated upper bound on the contribution of future terms to the probability is less than \texttt{tol}. There may also be some slight loss of accuracy due to calculation of gamma functions. For large values of \( \delta > 50 \) there may be significant loss of accuracy.
6.2 References

7 See Also
nag_prob_non_central_students_t (g01gbc)

8 Example
Values from, and degrees of freedom for and non-centrality parameter of the non-central Student’s t-
distributions are read, the lower tail probabilities calculated and all these values printed until the end of
data is reached.

8.1 Program Text
/* nag_prob_non_central_students_t (g01gbc) Example Program. 
  * Copyright 1999 Numerical Algorithms Group.
  * Mark 6, 2000.
  */
#include <stdio.h>
#include <nag.h>
#include <nag01.h>

int main(void)
{
  double delta, df, prob, t, tol;
  Integer max_iter;
  Integer exit_status = 0;
  NagError fail;
  INIT_FAIL(fail);
  Vprintf("g01gbc Example Program Results\n\n");
  /* Skip heading in data file */
  Vscanf("%*[\n]" surprisingly useful
  tol = 5e-6;
  max_iter = 50;
  while (!((scanf("%lf %lf %lf %*[\n]", &t, &df, &delta)) != EOF))
  {
    prob = g01gbc(t, df, delta, tol, max_iter, &fail);
    if (fail.code == NE_NOERROR)
      Vprintf("%8.3f%8.3f%8.3f%8.4f\n", t, df, delta, prob);
    else
      Vprintf("Error from g01gbc.\n\n", fail.message);
    exit_status=1;
    goto END;
  }
END:
  return exit_status;
}
8.2 Program Data

g01gbc Example Program Data
-1.528  20.0  2.0 :t df delta
-0.188  7.5   1.0 :t df delta
 1.138  45.0  0.0 :t df delta

8.3 Program Results

g01gbc Example Program Results

<table>
<thead>
<tr>
<th>t</th>
<th>df</th>
<th>delta</th>
<th>prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.528</td>
<td>20.000</td>
<td>2.000</td>
<td>0.0003</td>
</tr>
<tr>
<td>-0.188</td>
<td>7.500</td>
<td>1.000</td>
<td>0.1189</td>
</tr>
<tr>
<td>1.138</td>
<td>45.000</td>
<td>0.000</td>
<td>0.8694</td>
</tr>
</tbody>
</table>