NAG Library Routine Document

S30NAF

Note: before using this routine, please read the Users’ Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

1 Purpose

S30NAF computes the European option price given by Heston’s stochastic volatility model (see Heston (1993)).

2 Specification

SUBROUTINE S30NAF(CALPUT, M, N, X, S, T, SIGMAV, KAPPA, CORR, VAR0, ETA, GAMMA, R, Q, P, LDP, IFAIL)

INTEGER M, N, LDP, IFAIL

double precision X(M), S, T(N), SIGMAV, KAPPA, CORR, VAR0, ETA, GAMMA,
R, Q, P(LDP,N)

CHARACTER*1 CALPUT

3 Description

S30NAF computes the price of a European option using Heston’s stochastic volatility model. The return on the asset price, \( S \), is

\[
\frac{dS}{S} = (r - q) dt + \sqrt{v_t} dW^{(1)}_t
\]

and the instantaneous variance, \( v_t \), is defined by a mean-reverting square root stochastic process,

\[
dv_t = \kappa(\eta - v_t) dt + \sigma_v \sqrt{v_t} dW^{(2)}_t,
\]

where \( r \) is the risk-free annual interest rate; \( q \) is the annual dividend rate; \( \sigma_v \) is the volatility of the volatility, \( \sqrt{v_t} \); \( \kappa \) is the mean reversion rate; \( \eta \) is the long term variance. \( dW^{(i)}_t \), for \( i = 1, 2 \), denotes two correlated standard Brownian motions with

\[
\text{Cov}[dW^{(1)}_t, dW^{(2)}_t] = \rho dt.
\]

The option price is computed by evaluating the integral transform given by Lewis (2000) using the form of the characteristic function discussed by Albrecher et al. (2007), see also Kilin (2006). For a given strike price, \( X \), and time to expiry, \( T \), the price of a European call is

\[
P_{\text{call}} = S e^{-qT} - X e^{-rT} \frac{1}{\pi} \int_0^{\infty} e^{ikX} \tilde{H}(k, v, T) \frac{dk}{k^2 - ik}
\]

where \( \tilde{X} = \ln(S/X) + (r - q)T \) and

\[
\tilde{H}(k, v, t) = \exp \left( \frac{2k \eta}{\sigma_v^2} \left[ g - \ln \left( \frac{1 - be^{-h}}{1 - b} \right) \right] + v_t g \left[ \frac{1 - e^{-\eta}}{1 - b e^{-\eta}} \right] \right),
\]

\[
g = \frac{1}{2} (b - \xi),
\]

\[
h = \frac{b - \xi}{b + \xi},
\]

\[
\xi = \left[ b^2 + 4 \left( \frac{k^2 - ik}{\sigma_v^2} \right) \right]^{1/2},
\]

\[
b = \frac{1}{2} \left[ (1 - \gamma + ik) \rho \sigma_v + \sqrt{\kappa^2 - \gamma(1 - \gamma) \sigma_v^2} \right]
\]

with \( t = \sigma^2 T/2 \). Here \( \gamma \) is the risk aversion parameter of the representative agent with \( 0 \leq \gamma \leq 1 \).
value \( \gamma = 1 \) corresponds to \( \lambda = 0 \) (the market price of risk) in Heston’s paper (see Rouah and Vainberg (2007)).

The price of a put option is obtained by put-call parity.

4 References


Lewis A L (2000) Option Valuation Under Stochastic Volatility Finance Press, USA


5 Parameters

1: \( \text{CALPUT} \) – CHARACTER*1 \( \text{Input} \)

\( \text{On entry:} \) determines whether the option is a call or a put.

\( \text{CALPUT} = 'C' \)

A call. The holder has a right to buy.

\( \text{CALPUT} = 'P' \)

A put. The holder has a right to sell.

\( \text{Constraint:} \) \( \text{CALPUT} = 'C' \) or 'P'.

2: \( \text{M} \) – INTEGER \( \text{Input} \)

\( \text{On entry:} \) the number of strike prices to be used.

\( \text{Constraint:} \) \( \text{M} \geq 1. \)

3: \( \text{N} \) – INTEGER \( \text{Input} \)

\( \text{On entry:} \) the number of times to expiry to be used.

\( \text{Constraint:} \) \( \text{N} \geq 1. \)

4: \( X(\text{M}) \) – double precision array \( \text{Input} \)

\( \text{On entry:} \) \( X(i) \) must contain \( X_i \), the \( i \)th strike price, for \( i = 1, 2, \ldots, M. \)

\( \text{Constraint:} \) \( X(i) \geq z \) and \( X(i) \leq 1/z \), where \( z = \text{X02AMF()} \), the safe range parameter, for \( i = 1, 2, \ldots, M. \)

5: \( S \) – double precision \( \text{Input} \)

\( \text{On entry:} \) \( S \), the price of the underlying asset.

\( \text{Constraint:} \) \( S \geq z \) and \( S \leq 1/z \), where \( z = \text{X02AMF()} \), the safe range parameter.

6: \( T(\text{N}) \) – double precision array \( \text{Input} \)

\( \text{On entry:} \) \( T(i) \) must contain \( T_i \), the \( i \)th time, in years, to expiry, for \( i = 1, 2, \ldots, N. \)

\( \text{Constraint:} \) \( T(i) \geq z \), where \( z = \text{X02AMF()} \), the safe range parameter, for \( i = 1, 2, \ldots, N. \)
7: SIGMAV – double precision
   Input
   On entry: the volatility, $\sigma_v$, of the volatility process, $\sqrt{v_t}$. Note that a rate of 20% should be entered as 0.2.
   Constraint: SIGMAV > 0.0.

8: KAPPA – double precision
   Input
   On entry: $\kappa$, the long term mean reversion rate of the volatility.
   Constraint: KAPPA > 0.0.

9: CORR – double precision
   Input
   On entry: the correlation between the two standard Brownian motions for the asset price and the volatility.
   Constraint: $-1.0 \leq$ CORR $\leq 1.0$.

10: VAR0 – double precision
    Input
    On entry: the initial value of the variance, $v_1$, of the asset price.
    Constraint: VAR0 $\geq 0.0$.

11: ETA – double precision
    Input
    On entry: $\eta$, the long term mean of the variance of the asset price.
    Constraint: ETA $> 0.0$.

12: GAMMA – double precision
    Input
    On entry: the risk aversion parameter, $\gamma$, of the representative agent.
    Constraint: $0.0 \leq$ GAMMA $\leq 1.0$.

13: R – double precision
    Input
    On entry: $r$, the annual risk-free interest rate, continuously compounded. Note that a rate of 5% should be entered as 0.05.
    Constraint: $R \geq 0.0$.

14: Q – double precision
    Input
    On entry: $q$, the annual continuous yield rate. Note that a rate of 8% should be entered as 0.08.
    Constraint: $Q \geq 0.0$.

15: P[LDP,N] – double precision array
    Output
    On exit: the leading M and N part of the array P contains the computed option prices.

16: LDP – INTEGER
    Input
    On entry: the first dimension of the array P as declared in the (sub)program from which S30NAF is called.
    Constraint: $LDP \geq M$.

17: IFAIL – INTEGER
    Input/Output
    On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 2.3 in the Essential Introduction for details.
    On exit: IFAIL = 0 unless the routine detects an error (see Section 6).
For environments where it might be inappropriate to halt program execution when an error is
detected, the value –1 or 1 is recommended. If the output of error messages is undesirable, then
the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the
recommended value is 0. **When the value –1 or 1 is used it is essential to test the value of**
**IFAIL on exit.**

6 **Error Indicators and Warnings**

If on entry IFAIL = 0 or –1, explanatory error messages are output on the current error message unit (as
defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, CALPUT ≠ 'C' or 'P'.

IFAIL = 2

On entry, M ≤ 0.

IFAIL = 3

On entry, N ≤ 0.

IFAIL = 4

On entry, X(i) < z or X(i) > 1/z, where z = X02AMF(), the safe range parameter.

IFAIL = 5

On entry, S < z or S > 1/z, where z = X02AMF(), the safe range parameter.

IFAIL = 6

On entry, T(i) < z, where z = X02AMF(), the safe range parameter.

IFAIL = 7

On entry, SIGMAV ≤ 0.0.

IFAIL = 8

On entry, KAPPA ≤ 0.0.

IFAIL = 9

On entry, |CORR| > 1.0.

IFAIL = 10

On entry, VAR0 < 0.0.

IFAIL = 11

On entry, ETA ≤ 0.0.

IFAIL = 12

On entry, GAMMA < 0.0,

or GAMMA > 1.0.

IFAIL = 13

On entry, R < 0.0.
On entry, \( Q < 0.0 \).

IFAIL = 16

On entry, \( LDP < M \).

IFAIL = 17

Quadrature has not converged.

7 Accuracy

The accuracy of the output is determined by the accuracy of the numerical quadrature used to evaluate the integral in (1). An adaptive method is used which evaluates the integral to within a tolerance of \( \max(10^{-8}, 10^{-10} \times |I|) \), where \(|I|\) is the absolute value of the integral.

8 Further Comments

None.

9 Example

This example computes the price of a European call using Heston’s stochastic volatility model. The time to expiry is 6 months, the stock price is 100 and the strike price is 100. The risk-free interest rate is 5% per year, the volatility of the variance, \( \sigma_v \), is 22.5% per year, the mean reversion parameter, \( \kappa \), is 2.0, the long term mean of the variance, \( \eta \), is 0.01 and the correlation between the volatility process and the stock price process, \( \rho \), is 0.0.

9.1 Program Text

* S30NAF Example Program Text
* Mark 22 Release. NAG Copyright 2007.
* ...
* .. Parameters ..
INTEGER NIN, NOUT
PARAMETER (NIN=5,NOUT=6)
INTEGER LDP, MMAX, NMAX
PARAMETER (LDP=50,MMAX=50,NMAX=50)
* .. Local Scalars ..
DOUBLE PRECISION CORR, ETA, GAMMA, KAPPA, Q, R, S, SIGMAV, VAR0
INTEGER I, IFAIL, J, M, N
CHARACTER PUT
* .. Local Arrays ..
DOUBLE PRECISION P(LDP,NMAX), T(NMAX), X(MMAX)
* .. External Subroutines ..
EXTERNAL S30NAF
* .. Executable Statements ..
WRITE (NOUT,*) 'S30NAF Example Program Results'
WRITE (NOUT,*)
WRITE (NOUT,*) 'Heston’s Stochastic volatility Model'
* Skip heading in data file
READ (NIN,*)
READ (NIN,*) S, R, Q
READ (NIN,*) KAPPA, ETA, VARO, SIGMAV, CORR, GAMMA
READ M, N
READ (NIN,*) M, N
* IF (M.LE.MMAX .AND. N.LE.NMAX) THEN
READ array of strike/exercise prices, X
READ (NIN,*) (X(I),I=1,M)
READ (NIN,*) (T(I),I=1,N)
*
IFAIL = 1

* CALL S30NAF(PUT,M,N,X,S,T,SIGMAV,KAPPA,CORR,VARO,ETA,GAMMA,R,Q,
  + P,LDP,IFAIL)
*
IF (IFAIL.EQ.0) THEN
  IF (PUT.EQ.'C' .OR. PUT.EQ.'c') THEN
    WRITE (NOUT,*) 'European Call :
  ELSE IF (PUT.EQ.'P' .OR. PUT.EQ.'p') THEN
    WRITE (NOUT,*) 'European Put :
  END IF
  WRITE (NOUT,'(A,1X,F8.4)') ' Spot = ', S
  WRITE (NOUT,'(A,1X,F8.4)') ' Volatility of vol = ', SIGMAV
  WRITE (NOUT,'(A,1X,F8.4)') ' Mean reversion = ', KAPPA
  WRITE (NOUT,'(A,1X,F8.4)') ' Correlation = ', CORR
  WRITE (NOUT,'(A,1X,F8.4)') ' Variance = ', VARO
  WRITE (NOUT,'(A,1X,F8.4)') ' Mean of variance = ', ETA
  WRITE (NOUT,'(A,1X,F8.4)') ' Risk aversion = ', GAMMA
  WRITE (NOUT,'(A,1X,F8.4)') ' Rate = ', R
  WRITE (NOUT,'(A,1X,F8.4)') ' Dividend = ', Q
  ELSE
    WRITE (NOUT,*)
    WRITE (NOUT,99998) IFAIL
  END IF
END IF
*
99999 FORMAT (1X,2(F9.4,1X),6X,F9.4)
99998 FORMAT (1X,' ** S30AAF returned with IFAIL = ',I5)
END

9.2 Program Data

S30NAF Example Program Data

'C' : Call = 'C', Put = 'P'
100.0 0.05 0.0 : S, R, Q
2.0 0.01 0.01 0.225 0.0 1.0 : KAPPA, ETA, VARO, SIGMAV, CORR, GAMMA
1 1 : M, N
100.0 : X(I), I = 1,2,...N
0.5 : T(I), I = 1,2,...M

9.3 Program Results

S30NAF Example Program Results

Heston's Stochastic volatility Model
European Call :
Spot = 100.0000
Volatility of vol = 0.2250
Mean reversion = 2.0000
Correlation = 0.0000
Variance = 0.0100
Mean of variance = 0.0100
Risk aversion = 1.0000
<table>
<thead>
<tr>
<th>Strike</th>
<th>Expiry</th>
<th>Option Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>100.0000</td>
<td>0.5000</td>
<td>4.0851</td>
</tr>
</tbody>
</table>