1 Purpose
S30CCF computes the price of a binary or digital asset-or-nothing option.

2 Specification

SUBROUTINE S30CCF(CALPUT, M, N, X, S, T, SIGMA, R, Q, P, LDP, IFAIL)

INTEGER M, N, LDP, IFAIL
DOUBLE PRECISION X(M), S, T(N), SIGMA, R, Q, P(LDP,N)
CHARACTER*1 CALPUT

3 Description
S30CCF computes the price of a binary or digital asset-or-nothing option which pays the underlying asset itself, S, at expiration if the option is in-the-money (see Section 2.4 in the S Chapter Introduction). For a strike price, X and underlying asset price S, and time to expiry T, the payoff is therefore S if S > X for a call or S < X for a put. Nothing is paid out when this condition is not met.

The price of a call with volatility, \sigma, risk-free interest rate, r, and annualised dividend yield, q, is

\[ P_{\text{call}} = S e^{-qT} \Phi(d_1) \]

and for a put,

\[ P_{\text{put}} = S e^{-qT} \Phi(-d_1) \]

\( \Phi \) is the cumulative Normal distribution function,

\[ \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp \left( -\frac{y^2}{2} \right) dy, \]

and

\[ d_1 = \frac{\ln(S/X) + (r-q+\sigma^2/2)T}{\sigma \sqrt{T}}. \]

4 References
Reiner E and Rubinstein M (1991) Unscrambling the binary code Risk 4

5 Parameters

1: CALPUT – CHARACTER*1

On entry: determines whether the option is a call or a put.

CALPUT = 'C'
A call. The holder has a right to buy.

CALPUT = 'P'
A put. The holder has a right to sell.

Constraint: CALPUT = 'C' or 'P'.
2: M – INTEGER
   *Input*
   
   On entry: the number of strike prices to be used.
   
   Constraint: M ≥ 1.

3: N – INTEGER
   *Input*
   
   On entry: the number of times to expiry to be used.
   
   Constraint: N ≥ 1.

4: X(M) – double precision array
   *Input*
   
   On entry: X(i) must contain X_i, the i_th strike price, for i = 1, 2, ..., M.
   
   Constraint: X(i) ≥ z and X(i) ≤ 1/z, where z = X02AMF(), the safe range parameter, for i = 1, 2, ..., M.

5: S – double precision
   *Input*
   
   On entry: S, the price of the underlying asset.
   
   Constraint: S ≥ z and S ≤ 1/z, where z = X02AMF(), the safe range parameter.

6: T(N) – double precision array
   *Input*
   
   On entry: T(i) must contain T_i, the i_th time, in years, to expiry, for i = 1, 2, ..., N.
   
   Constraint: T(i) ≥ z, where z = X02AMF(), the safe range parameter, for i = 1, 2, ..., N.

7: SIGMA – double precision
   *Input*
   
   On entry: σ, the volatility of the underlying asset. Note that a rate of 15% should be entered as 0.15.
   
   Constraint: SIGMA > 0.0.

8: R – double precision
   *Input*
   
   On entry: r, the annual risk-free interest rate, continuously compounded. Note that a rate of 5% should be entered as 0.05.
   
   Constraint: R ≥ 0.0.

9: Q – double precision
   *Input*
   
   On entry: q, the annual continuous yield rate. Note that a rate of 8% should be entered as 0.08.
   
   Constraint: Q ≥ 0.0.

10: P(LDP,N) – double precision array
    *Output*
    
    On exit: the leading M and N part of the array P contains the computed option prices.

11: LDP – INTEGER
    *Input*
    
    On entry: the first dimension of the array P as declared in the (sub)program from which S30CCF is called.
    
    Constraint: LDP ≥ M.

12: IFAIL – INTEGER
    *Input/Output*
    
    On entry: IFAIL must be set to 0, −1 or 1. If you are unfamiliar with this parameter you should refer to Section 2.3 in the Essential Introduction for details.
    
    On exit: IFAIL = 0 unless the routine detects an error (see Section 6).
For environments where it might be inappropriate to halt program execution when an error is detected, the value $-1$ or $1$ is recommended. If the output of error messages is undesirable, then the value $1$ is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is $0$. **When the value $-1$ or $1$ is used it is essential to test the value of IFAIL on exit.**

### 6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or $-1$, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

- **IFAIL $= 1$**
  - On entry, CALPUT $\neq 'C'$ or 'P'.

- **IFAIL $= 2$**
  - On entry, $M \leq 0$.

- **IFAIL $= 3$**
  - On entry, $N \leq 0$.

- **IFAIL $= 4$**
  - On entry, $X(i) < z$ or $X(i) > 1/z$, where $z = X02AMF()$, the safe range parameter.

- **IFAIL $= 5$**
  - On entry, $S < z$ or $S > 1/z$, where $z = X02AMF()$, the safe range parameter.

- **IFAIL $= 6$**
  - On entry, $T(i) < z$, where $z = X02AMF()$, the safe range parameter.

- **IFAIL $= 7$**
  - On entry, SIGMA $\leq 0.0$.

- **IFAIL $= 8$**
  - On entry, $R < 0.0$.

- **IFAIL $= 9$**
  - On entry, $Q < 0.0$.

- **IFAIL $= 11$**
  - On entry, LDP $< M$.

### 7 Accuracy

The accuracy of the output is dependent on the accuracy of the cumulative Normal distribution function, $\Phi$. This is evaluated using a rational Chebyshev expansion, chosen so that the maximum relative error in the expansion is of the order of the **machine precision** (see S15ABF and S15ADF). An accuracy close to **machine precision** can generally be expected.

### 8 Further Comments

None.
9 Example

This example computes the price of an asset-or-nothing put with a time to expiry of 0.5 years, a stock price of 70 and a strike price of 65. The risk-free interest rate is 7% per year, there is an annual dividend return of 5% and the volatility is 27% per year.

9.1 Program Text

* S30CCF Example Program Text
* Mark 22 Release. NAG Copyright 2008.
* .. Parameters ..
  INTEGER NIN, NOUT
  PARAMETER (NIN=5, NOUT=6)
  INTEGER LDP, MMAX, NMAX
  PARAMETER (LDP=50, MMAX=50, NMAX=50)
* .. Local Scalars ..
  DOUBLE PRECISION Q, R, S, SIGMA
  INTEGER I, IFAIL, J, M, N
  CHARACTER PUT
* .. Local Arrays ..
  DOUBLE PRECISION P(LDP,NMAX), T(NMAX), X(MMAX)
* .. External Subroutines ..
  EXTERNAL S30CCF
* .. Executable Statements ..
  WRITE (NOUT,*) 'S30CCF Example Program Results'
  WRITE (NOUT,*)
  WRITE (NOUT,*) 'Binary (Digital): Asset-or-Nothing'
  * Skip heading in data file
  READ (NIN,*)
  * Read problem parameters
  READ (NIN,*) PUT
  READ (NIN,*) S, SIGMA, R, Q
  READ (NIN,*) M, N
  IF (M.LE.MMAX .AND. N.LE.NMAX) THEN
    * Read array of strike/exercise prices, X
    READ (NIN,*) (X(I), I=1, M)
    READ (NIN,*) (T(I), I=1, N)
    IFAIL = 1
  ELSE
    WRITE (NOUT,*)
    WRITE (NOUT,99998) IFAIL
  END IF
  CALL S30CCF(PUT, M, N, X, S, T, SIGMA, R, Q, P, LDP, IFAIL)
  IF (IFAIL.EQ.0) THEN
    IF (PUT.EQ.'C' .OR. PUT.EQ.'c') THEN
      WRITE (NOUT,*) 'European Call :'
    ELSE IF (PUT.EQ.'P' .OR. PUT.EQ.'p') THEN
      WRITE (NOUT,*) 'European Put :'
    END IF
    WRITE (NOUT,*) ' Spot = ', S
    WRITE (NOUT,*) ' Volatility = ', SIGMA
    WRITE (NOUT,*) ' Rate = ', R
    WRITE (NOUT,*) ' Dividend = ', Q
  END IF
  WRITE (NOUT,*) ' Strike Expiry Option Price'
  DO 40 I = 1, M
    DO 20 J = 1, N
      WRITE (NOUT,99999) X(I), T(J), P(I,J)
  20 CONTINUE
  40 CONTINUE
  ELSE
    WRITE (NOUT,99999) IFAIL
  END IF
END IF

* 
99999 FORMAT (1X,2(F9.4,1X),6X,F9.4)
99998 FORMAT (1X,’ ** S30CCF returned with IFAIL = ’,I5)
END

9.2 Program Data

S30CCF Example Program Data
’P’ : Call = ‘C’, Put = ‘P’
70.0 0.27 0.07 0.05 : S, SIGMA, R, Q
1 1 : M, N
65.0 : X(I), I = 1,2,...M
0.5 : T(I), I = 1,2,...N

9.3 Program Results

S30CCF Example Program Results

Binary (Digital): Asset-or-Nothing
European Put :
Spot = 70.0000
Volatility = 0.2700
Rate = 0.0700
Dividend = 0.0500

<table>
<thead>
<tr>
<th>Strike</th>
<th>Expiry</th>
<th>Option Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>65.0000</td>
<td>0.5000</td>
<td>20.2069</td>
</tr>
</tbody>
</table>