1 Purpose
S30CAF computes the price of a binary or digital cash-or-nothing option.

2 Specification
SUBROUTINE S30CAF(CALPUT, M, N, X, S, K, T, SIGMA, R, Q, P, LDP, IFAIL)
   INTEGER M, N, LDP, IFAIL
   double precision X(M), S, K, T(N), SIGMA, R, Q, P(LDP,N)
   CHARACTER*1 CALPUT

3 Description
S30CAF computes the price of a binary or digital cash-or-nothing option which pays a fixed amount, K, at expiration if the option is in-the-money (see Section 2.4 in the S Chapter Introduction). For a strike price, X and underlying asset price S, and time to expiry T, the payoff is therefore K if S > X for a call or S < X for a put. Nothing is paid out when this condition is not met.

The price of a call with volatility, σ, risk-free interest rate, r, and annualised dividend yield, q, is

\[ P_{\text{call}} = Ke^{-rT} \phi(d_2) \]

and for a put,

\[ P_{\text{put}} = Ke^{-rT} \phi(-d_2) \]

\( \phi \) is the cumulative Normal distribution function,

\[ \phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp \left( -\frac{y^2}{2} \right) dy, \]

and

\[ d_2 = \frac{\ln(S/X) + (r - q - \sigma^2/2)T}{\sigma \sqrt{T}}. \]

4 References
Reiner E and Rubinstein M (1991) Unscrambling the binary code Risk 4

5 Parameters
1: CALPUT – CHARACTER*1
   Input
   On entry: determines whether the option is a call or a put.
   CALPUT = 'C'
      A call. The holder has a right to buy.
   CALPUT = 'P'
      A put. The holder has a right to sell.
   Constraint: CALPUT = 'C' or 'P'.

Note: before using this routine, please read the Users’ Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.
2: M – INTEGER  
   *Input*  
   *On entry:* the number of strike prices to be used.  
   *Constraint:* \( M \geq 1 \).

3: N – INTEGER  
   *Input*  
   *On entry:* the number of times to expiry to be used.  
   *Constraint:* \( N \geq 1 \).

4: X(M) – *double precision* array  
   *Input*  
   *On entry:* \( X(i) \) must contain \( X_i \), the \( i \)th strike price, for \( i = 1, 2, \ldots, M \).  
   *Constraint:* \( X(i) \geq z \) and \( X(i) \leq 1/z \), where \( z = X02AMF() \), the safe range parameter, for \( i = 1, 2, \ldots, M \).

5: S – *double precision*  
   *Input*  
   *On entry:* \( S \), the price of the underlying asset.  
   *Constraint:* \( S \geq z \) and \( S \leq 1/z \), where \( z = X02AMF() \), the safe range parameter.

6: K – *double precision*  
   *Input*  
   *On entry:* the amount to be paid at expiration if the option is in-the-money, i.e., if \( S > X \) when \( \text{CALPUT} = 'C' \), or if \( S < X \) when \( \text{CALPUT} = 'P' \).  
   *Constraint:* \( K \geq 0.0 \).

7: T(N) – *double precision* array  
   *Input*  
   *On entry:* \( T(i) \) must contain \( T_i \), the \( i \)th time, in years, to expiry, for \( i = 1, 2, \ldots, N \).  
   *Constraint:* \( T(i) \geq z \), where \( z = X02AMF() \), the safe range parameter, for \( i = 1, 2, \ldots, N \).

8: SIGMA – *double precision*  
   *Input*  
   *On entry:* \( \sigma \), the volatility of the underlying asset. Note that a rate of 15% should be entered as 0.15.  
   *Constraint:* \( \text{SIGMA} > 0.0 \).

9: R – *double precision*  
   *Input*  
   *On entry:* \( r \), the annual risk-free interest rate, continuously compounded. Note that a rate of 5% should be entered as 0.05.  
   *Constraint:* \( R \geq 0.0 \).

10: Q – *double precision*  
    *Input*  
    *On entry:* \( q \), the annual continuous yield rate. Note that a rate of 8% should be entered as 0.08.  
    *Constraint:* \( Q \geq 0.0 \).

11: P(LDP,N) – *double precision* array  
    *Output*  
    *On exit:* the leading \( M \) and \( N \) part of the array \( P \) contains the computed option prices.

12: LDP – INTEGER  
    *Input*  
    *On entry:* the first dimension of the array \( P \) as declared in the (sub)program from which S30CAF is called.  
    *Constraint:* \( LDP \geq M \).
13: IFAIL – INTEGER

On entry: IFAIL must be set to 0, −1 or 1. If you are unfamiliar with this parameter you should refer to Section 2.3 in the Essential Introduction for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value −1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. When the value −1 or 1 is used it is essential to test the value of IFAIL on exit.

6 Error Indicators and Warnings

If on entry IFAIL = 0 or −1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1
On entry, CALPUT ≠ 'C' or 'P'.

IFAIL = 2
On entry, M ≤ 0.

IFAIL = 3
On entry, N ≤ 0.

IFAIL = 4
On entry, X(i) < z or X(i) > 1/z, where z = X02AMF(), the safe range parameter.

IFAIL = 5
On entry, S < z or S > 1/z, where z = X02AMF(), the safe range parameter.

IFAIL = 6
On entry, K < 0.0.

IFAIL = 7
On entry, T(i) < z, where z = X02AMF(), the safe range parameter.

IFAIL = 8
On entry, SIGMA ≤ 0.0.

IFAIL = 9
On entry, R < 0.0.

IFAIL = 10
On entry, Q < 0.0.

IFAIL = 12
On entry, LDP < M.
7 Accuracy

The accuracy of the output is dependent on the accuracy of the cumulative Normal distribution function, $\Phi$. This is evaluated using a rational Chebyshev expansion, chosen so that the maximum relative error in the expansion is of the order of the machine precision (see S15ABF and S15ADF). An accuracy close to machine precision can generally be expected.

8 Further Comments

None.

9 Example

This example computes the price of a cash-or-nothing put with a time to expiry of 0.75 years, a stock price of 100 and a strike price of 80. The risk-free interest rate is 6% per year and the volatility is 35% per year. If the option is in-the-money at expiration, i.e., if $S > X$, the payoff is 10.

9.1 Program Text

* S30CAF Example Program Text
* Mark 22 Release. NAG Copyright 2008.
* .. Parameters ..
INTEGER NIN, NOUT
PARAMETER (NIN=5,NOUT=6)
INTEGER LDP, MMAX, NMAX
PARAMETER (LDP=50,MMAX=50,NMAX=50)
* .. Local Scalars ..
DOUBLE PRECISION K, Q, R, S, SIGMA
INTEGER I, IFAIL, J, M, N
CHARACTER PUT
* .. Local Arrays ..
DOUBLE PRECISION P(LDP,NMAX), T(NMAX), X(MMAX)
* .. External Subroutines ..
EXTERNAL S30CAF
* .. Executable Statements ..
WRITE (NOUT,*) 'S30CAF Example Program Results'
WRITE (NOUT,*)
WRITE (NOUT,*) 'Binary (Digital): Cash-or-Nothing'
* Skip heading in data file
READ (NIN,*)
* Read problem parameters
READ (NIN,*) PUT
READ (NIN,*) S, K, SIGMA, R, Q
READ (NIN,*) M ,N
* IF (M.LE.MMAX .AND. N.LE.NMAX) THEN
* Read array of strike/exercise prices, X
READ (NIN,*) (X(I),I=1,M)
READ (NIN,*) (T(I),I=1,N)
* IFAIL = 1
* CALL S30CAF(PUT,M,N,X,S,K,T,SIGMA,R,Q,P,LDP,IFAIL)
* IF (IFAIL.EQ.0) THEN
IF (PUT.EQ.‘C’ .OR. PUT.EQ.‘c’) THEN
WRITE (NOUT,’(A,1X,F8.4)’) ‘European Call :’
ELSE IF (PUT.EQ.‘P’ .OR. PUT.EQ.‘p’) THEN
WRITE (NOUT,’(A,1X,F8.4)’) ‘European Put :
END IF
WRITE (NOUT,’(A,1X,F8.4)’) ‘ Spot = ’, S
WRITE (NOUT,’(A,1X,F8.4)’) ‘ Payout = ’, K
WRITE (NOUT,’(A,1X,F8.4)’) ‘ Volatility = ’, SIGMA
WRITE (NOUT,’(A,1X,F8.4)’) ‘ Rate = ’, R
WRITE (NOUT,’(A,1X,F8.4)’) ‘ Dividend = ’, Q
WRITE (NOUT,*)
WRITE (NOUT,*') ' Strike  Expiry  Option Price'
DO 40 I = 1, M
   DO 20 J = 1, N
      WRITE (NOUT,99999) X(I), T(J), P(I,J)
   CONTINUE
40 CONTINUE
ELSE
   WRITE (NOUT,*)
   WRITE (NOUT,99998) IFAIL
END IF
END

* 
99999 FORMAT (1X,2(F9.4,1X),6X,F9.4)
99998 FORMAT (1X,' ** S30CAF returned with IFAIL = ',I5)
END

9.2 Program Data

S30CAF Example Program Data
 'P' : Call = 'C', Put = 'P'
 100.0 10.0 0.35 0.06 0.0 : S, K, SIGMA, R, Q
 1 1 : M, N
 80.0 : X(I), I = 1,2,...M
 0.75 : T(I), I = 1,2,...N

9.3 Program Results

S30CAF Example Program Results

Binary (Digital): Cash-or-Nothing
European Put :
Spot    = 100.0000
Payout  = 10.0000
Volatility = 0.3500
Rate    = 0.0600
Dividend = 0.0000

Strike  Expiry  Option Price
80.0000  0.7500  2.2155